# Mathematical Analysis

In this section, we first prove there is no over-estimation error for flows stored in the filter layer, and then derive the formula of the error bound.

## Proof of No Over-estimation Error

**Theorem 1:** let  and  be the real frequency and estimated frequency of flow  at time , and assume  is the mapped slot in filter layer,  represents the value of pvote at time . Then



**Proof.** We use mathematical induction to prove. When , no packet maps into this bucket before. Therefore,  and . The theorem holds. If the theorem holds, mean when , then we must have  when . Next, we prove the theorem holds when .

For the incoming packet, there are four cases as follows:

Case1:  is not recorded in slot . Obviously,  and the theorem holds.

Case2:  is recorded in slot , and the incoming packet isn't mapped to . Then the pvote in  don't increase, so the theorem holds.

Case3:  is recorded in slot , and the incoming packet mapped to  but does not belong to flow . The nvote will increase with a certain probability. If nvote is larger than pvote, then . Otherwise, .

Case4:  is recorded in slot , and the incoming packet mapped to  belonging to flow . The pvote increases by 1, and  .In the meantime,  .The theorem holds.

In summary, the theorem holds at any point of time.

## The Error Bound in HeavySeparation

**Theorem 2:** Assume  is the -th elephant flow sorting based on frequency. And assume that  is always recorded as a candidate flow. In other words, when a data packet belonging to  arrives,  is either already recorded in a certain slot, or there is an empty slot. Given a positive number , we have



Where .

**Proof.** According to the operation of HeavySeparation, for each incoming packet belonging to , its estimated frequency is incremented by 1. The estimated frequency decreases to 0 only when the nvote is equal to pvote. So, we analyze the situation of nvote increasing.

We define a variable  represent the sum of the sizes of other distinct flows which are mapped to the same slot with  and successfully decrease the pvote of corresponding slot. Then we have . First, let V be the number of packets not belonging to , then the expectation of V is:



Second, if the nvote increases only when the pvote of  is the smallest in the bucket. In other words, there are  larger elephant flows mapped to this bucket. Then we define  to represent the probability of taking the slot of  as the smallest slot.



Third, as described above, the nvote in slot increase using the power-weakening increment method, the probability is:



Because we have assumed that  is always recorded as a candidate flow. Thus, the estimated frequency  is the value of pvote in the corresponding slot. We assume that the power increment occurs randomly as the estimated frequency grows from 1 to . Then we get the expectation:



Then we get the expectation of :



Based on Markov inequality, we have:

